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Semiclassical theory of short periodic orbits in quantum chaos

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Abstract. We have developed a semiclassical theory of short periodic orbits to obtain all the quantum information of a bounded chaotic Hamiltonian system. If T_1 is the period of the shortest periodic orbit, T_2 the period of the next one and so on, the number N_{po} of periodic orbits required in the calculation is such that $T_1 + \dots + T_{N_{po}} \simeq T_H$, with T_H the Heisenberg time. As a result $N_{po} \simeq hT_H / \ln(hT_H)$, where h is the topological entropy. For methods related to the trace formula $N_{po} \simeq \exp(hT_H) / (hT_H)$.

The semiclassical evaluation of energy spectra in classically chaotic Hamiltonian systems was started in 1971 with Gutzwiller's trace formula [1]. From then on great effort has been dedicated to extend the formalism to wavefunctions [2] and to use resummation techniques to improve the convergence properties of the trace formulae [3, 4]. However, a common drawback in all of these approaches is the requirement of an enormous number of periodic orbits (POs), restricting explicit calculations to very special systems; e.g. systems where the classical mechanics is handled by a symbolic dynamics. Moreover, these approaches miss a simple understanding of wave mechanics in terms of classical objects.

Recently, it was shown that chaotic eigenfunctions can be described in terms of a small number of localized structures living on short periodic orbits [5]. Having this in mind, we present a simple semiclassical formalism to obtain quantum mechanics from a very small number of short POs. In order to do so, we will construct resonances (or scar functions)[†] of short POs explicitly, evaluating the interaction between them.

Let γ be an unstable PO isolated on each energy surface. We introduce a curvilinear coordinate system choosing the x -axis along the trajectory and the y -axis perpendicular to it at x (to simplify the exposition we take only one transverse direction). On the PO $y = 0$. Classical mechanics in a neighbourhood of the orbit is governed by a transverse symplectic matrix $M(x)$ of elements $m_{ij}(x)$ ($i, j = 1, 2$), which describes the linearized motion on the energy surface. Then, a point with transverse coordinates (y, p_y) at $x = 0$ evolves according to the following rule:

$$\begin{pmatrix} y(x) \\ p_y(x) \end{pmatrix} = \begin{pmatrix} m_{11}(x) & m_{12}(x) \\ m_{21}(x) & m_{22}(x) \end{pmatrix} \begin{pmatrix} y \\ p_y \end{pmatrix}.$$

[†] Recently [6], a semiclassical elementary construction on the boundary of billiards was presented, which associates a function to each periodic orbit. These functions were called *scar functions*. In this work we prefer to use the name *resonances* because the constructive method only depends on the orbit and its neighbourhood and can also be applied to open systems.

We have selected the origin $x = 0$ such that $m_{11}(L) = m_{22}(L)$, with L the length of γ^\dagger . There are at least 2ν points on the orbit satisfying that condition, with ν the Maslov index. With this choice the monodromy matrix $M(L)$ acquires the form,

$$M(L) = (-1)^\nu \begin{pmatrix} \cosh(\lambda L) & \sinh(\lambda L)/\tan(\varphi) \\ \sinh(\lambda L)\tan(\varphi) & \cosh(\lambda L) \end{pmatrix}$$

where λ is the Lyapunov exponent in units of $[\text{length}^{-1}]$ and $\tan(\varphi) (\neq 0)$ in units of $[\text{momentum}/\text{length}]$ defines the slope of the unstable manifold in the plane $y-p_y$ (where the slope of the stable manifold is $-\tan(\varphi)$).

In general, it is impossible to compare vectors living in the plane $y-p_y$ because the axes have different units. However, when the directions are symmetrical with respect to the axes it is only necessary to compare one component. Then, we change to new axes ξ_u and ξ_s , on the unstable and stable manifolds, respectively, such that their projections on each axis are equal in absolute value. The symplectic matrix B transforming coordinates from the new axes into the old ones is

$$B = (\xi_u \ \xi_s) = (1/\sqrt{2}) \begin{pmatrix} 1/\alpha & -s/\alpha \\ s\alpha & \alpha \end{pmatrix}$$

with $\alpha = \sqrt{|\tan(\varphi)|}$ and $s = \text{sign}(\varphi)$. Observe that $B^{-1}M(L)B = (-1)^\nu \exp(\lambda LD)$, with D a diagonal matrix of elements $d_{11} = 1$ and $d_{22} = -1$.

Now, we decompose $M(x)$ into a periodic matrix $F(x)$ describing the evolution of the manifolds, and a matrix (depending in a simple way on x) describing the exponential contraction–dilation along the manifolds,

$$M(x) = F(x) \exp(x\lambda K) \equiv F(x)B \exp(x\lambda D) B^{-1} \quad (1)$$

with $K \equiv BDB^{-1}$ ($k_{11} = k_{22} = 0$, $k_{21} = 1/k_{12} = \tan(\varphi)$). Equation (1) defines $F(x)$ in terms of $M(x)$ and we can see that $F(L) = (-1)^\nu 1$. Floquet's theorem [7] affirms that the decomposition given in (1) can also be obtained for systems with many transverse directions.

Now, it is possible to construct a family of resonances associated with γ . We adapt well known semiclassical techniques to obtain eigenfunctions concentrated in the neighbourhood of stable POs [8]. In the x -direction we consider the typical solution of a one-dimensional motion, and in the transverse one we use a wavepacket which evolves according to $F(x)$,

$$\psi_\gamma(x, y) = \frac{\exp\{i[S(x) + y^2 \Gamma(x)/2]/\hbar - i\phi(x)/2\}}{\sqrt{T\dot{x}} [\pi(\hbar/J)|Q(x)|^2]^{1/4}} \quad (2)$$

where $S(x) = \int_0^x m\dot{x} dx$ is the action, T the period of γ and J the unit area in the plane $y-p_y$. $\Gamma(x) \equiv P(x)/Q(x)$, with $Q(x)$ ($P(x)$) the y (p_y) component of the complex vector ‡ .

$$\xi_u(x) + i\xi_s(x) \equiv F(x)(\xi_u + i\xi_s) = M(x)B \begin{pmatrix} e^{-x\lambda} \\ i e^{x\lambda} \end{pmatrix}. \quad (3)$$

Equation (3) shows that it is not necessary to evaluate explicitly $F(x)$. On the other hand, the area-preserving property of $F(x)$ guarantees the following normalization condition:

$$Q^*(x) P(x) - Q(x) P^*(x) = 2i\xi_u(x) \wedge \xi_s(x) = 2iJ. \quad (4)$$

† The monodromy matrix starting at $x = x_0$ is related to $M(x)$ in a simple way: $M_{x_0}(L) = M(x_0)M(L)M(x_0)^{-1}$. So, it is easy to find the required condition.

‡ The initial complex vector $\xi_u + i\xi_s$ defines at $x = 0$ a transverse wavepacket with the following two properties: (a) maximum overlap with the wavepacket obtained from it by a one-period linealized evolution, and (b) maximum concentration around the orbit minimizing non-linear contributions to the one-period exact evolution.

Then, $\text{Im}[\Gamma(x)] = J/|Q(x)|^2 > 0$. Accordingly, ψ_γ is concentrated around γ and well behaved in all regions except in the neighbourhood of a turning point ($\dot{x} = 0$)[†]. Finally, the complex number $Q(x)$ sweeps an angle $\phi(x)$ while evolving from 0 to x , and $\nu \equiv \phi(L)/\pi$ (this is the definition of the Maslov index).

ψ_γ is a continuous function at $x = L$ if the accumulated phase around the orbit is an integral multiple of 2π . This condition determines the admitted energies E_γ of the PO and corresponds to the Bohr–Sommerfeld quantization rule: $S(L)/\hbar - \nu\pi/2 = 2n\pi$, where $n = 0, 1, \dots$, is the number of excitations along γ .

We stress that the semiclassical construction of eigenfunctions in the neighbourhood of *stable* orbits is similar to (2). The initial complex vector of equation (3) is replaced by the eigenvector of the monodromy matrix (a complex vector in this case) satisfying (4). And of course, the evolution of the vector is given by the transverse symplectic matrix without modifications. Eigenvalues have an error $\mathcal{O}(\hbar)$ and eigenfunctions an error $\mathcal{O}(\sqrt{\hbar})$. Moreover, it is possible to improve the accuracy by including transverse excitations [8].

In our case, there is an essential error because the evolution in (3) is given by a modified transverse matrix. In order to eliminate that error, we will first evaluate the action of the semiclassical evolution operator for infinitesimal times over the resonance in the form

$$\hat{H} \equiv i\hbar \lim_{\delta t \rightarrow 0} (\hat{U}(\delta t) - \hat{1})/\delta t. \tag{5}$$

The classical transverse evolution from x to $x + \delta x$ is given by $M_x(\delta x) = M(x + \delta x) M(x)^{-1}$. So, $M_x(\delta x) F(x) = F(x + \delta x) \exp(\delta x \lambda K)$. Observing that $K \xi_u = \xi_u$ and $K \xi_s = -\xi_s$, it results to first order in δx :

$$\begin{aligned} M_x(\delta x) \xi_u(x) &\simeq (1 + \delta x \lambda) \xi_u(x + \delta x) \\ M_x(\delta x) \xi_s(x) &\simeq (1 - \delta x \lambda) \xi_s(x + \delta x). \end{aligned} \tag{6}$$

The above expressions actually show clearly the approximation involved in the construction. We have forced the vector ξ_u (ξ_s) to evolve without dilation (contraction), while the right evolution dilates (contracts) the vector with a rate specified by λ . Using (6) we see immediately that the periodic functions $Q(x)$ and $P(x)$ are transformed into $\tilde{Q}(x) = Q(x) + \delta x \lambda Q(x)^*$ and $\tilde{P}(x) = P(x) + \delta x \lambda P(x)^*$, respectively. Then, the application of the semiclassical evolution operator to the resonance gives

$$\hat{U}(\delta t = \delta x/\dot{x}) \psi_\gamma(x, y) = e^{-iE_\gamma \delta t/\hbar} \tilde{\psi}_\gamma(x, y) \tag{7}$$

where $\tilde{\psi}_\gamma(x, y)$ has the expression given in (2) but using the transformed functions $\tilde{Q}(x)$ and $\tilde{P}(x)$. Finally, by replacing (7) into (5), we obtain

$$\hat{H} \psi_\gamma(x, y) = g_\gamma(x, y) \psi_\gamma(x, y) \tag{8}$$

with $g_\gamma(x, y) = E_\gamma + i\hbar \dot{x} \lambda (y^2 J/\hbar - |Q(x)|^2/2)/Q(x)^2$.

Then, the application of the semiclassical Hamiltonian operator to the resonance gives the term $E_\gamma \psi_\gamma$ as expected, plus a resonance of γ with two excitations in the transverse direction. In fact, the two excitations are also expected because the right evolution produces a quadrupole-like deformation of the wavepacket. Equation (8) is an extremely powerful tool: it is the key to evaluating matrix elements between POs. As the operator \hat{H} is not exactly Hermitian for finite values of \hbar , we define a symmetrized interaction between two POs γ and δ as follows (in Dirac notation):

$$\begin{aligned} \langle \delta | \hat{H} | \gamma \rangle &\equiv (\langle \delta | \hat{H} | \gamma \rangle + \langle \gamma | \hat{H} | \delta \rangle^*)/2 \\ \langle \delta | \hat{H}^2 | \gamma \rangle &\equiv \langle \hat{H} \delta | \hat{H} \gamma \rangle. \end{aligned} \tag{9}$$

[†] An expression in that region can be obtained by using the Airy function. There are no problems if the turning point occurs in a hard-wall potential.

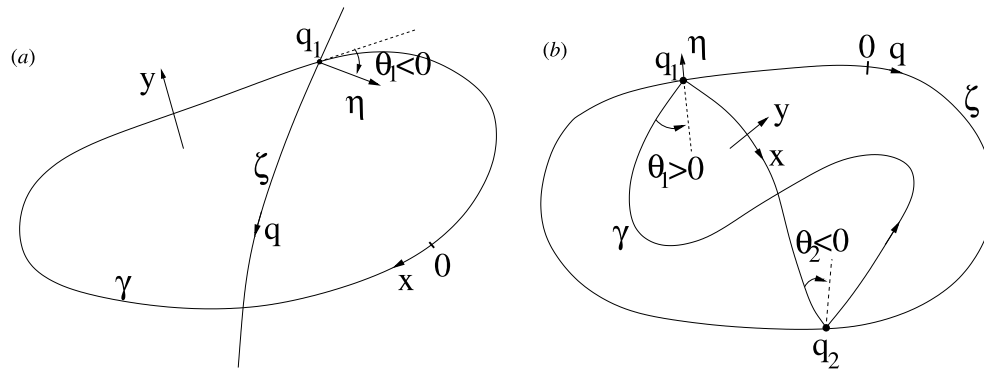


Figure 1. (a) Coordinates (x, y) and (q, η) defining the neighbourhood of the periodic orbit γ and the section ζ , respectively. (b) As in (a) but the section is given by a hard wall.

By using (8), we obtain explicitly the following diagonal matrix elements in the semiclassical limit $\hbar \rightarrow 0$,

- (a) $\langle \gamma | \gamma \rangle \rightarrow 1$
- (b) $\bar{E}_\gamma \equiv \langle \gamma | \hat{H} | \gamma \rangle / \langle \gamma | \gamma \rangle \rightarrow E_\gamma$
- (c) $\sigma_\gamma^2 \equiv \langle \gamma | \hat{H}^2 | \gamma \rangle / \langle \gamma | \gamma \rangle - \bar{E}_\gamma^2 \rightarrow (\hbar\lambda)^2 \bar{x}^2 / 2$

where $\bar{x}^2 = S(L)/mT$ is the time average of \dot{x}^2 on the orbit. Expression (c) shows that the width σ_γ of the resonance is asymptotically proportional to λ . Moreover, $\rho_E \sigma_\gamma = \mathcal{O}(\hbar^{-1})$ [†] shows that a unique orbit cannot support a stationary state in the semiclassical limit (to support an eigenfunction the width of a resonance needs to satisfy $\rho_E \sigma < 1$). Of course, this result is well known [9].

We should say something about symmetry. If the system can be time reversed it is possible to show that $\psi_{-\gamma}(x, y) = \psi_\gamma(x, y)^*$ and $\hat{H}\psi_{-\gamma}(x, y) = (\hat{H}\psi_\gamma(x, y))^*$, where $-\gamma$ is the time reversal partner of γ . If the system also includes a spatial symmetry G , it results that $\psi_{G\gamma}(x, y) = G\psi_\gamma(x, y)$ and $\hat{H}\psi_{G\gamma}(x, y) = G\hat{H}\psi_\gamma(x, y)$. So, to obtain real eigenfunctions inside a defined symmetry representation, we construct real resonances inside the same representation by using group theory [10].

For low or medium energies we can evaluate matrix elements directly on the domain; however, for high energies or to obtain explicit expressions in terms of classical quantities as \hbar goes to zero, it is preferable to work on a surface of section [11]. Let ζ be a differentiable curve with the coordinate q along it, and η perpendicular to ζ at q ($\eta = 0$ on the curve). Suppose γ crosses the section at q_j ($j = 1, \dots, m$) with angles θ_j ; the corresponding positions on γ are x_j (see figure 1(a)). In a neighbourhood of radius $\mathcal{O}(\sqrt{\hbar})$ around the intersection point j , the coordinates are related to order \hbar by $x - x_j = \sin(\theta_j)(q - q_j) - \cos(\theta_j)\eta$ and $y = \cos(\theta_j)(q - q_j) + \sin(\theta_j)\eta$. Then, the restriction of ψ_γ to ζ , up to $\mathcal{O}(\sqrt{\hbar})$, is a sum of wavepackets in one dimension with tangential momentum $p_j = m\dot{x}_j \sin(\theta_j)$,

$$\varphi_\gamma(q) \equiv \psi_\gamma(x, y)|_\zeta = \sum_{j=1}^m \psi_{\gamma_j}(q) \quad (10)$$

where $\psi_{\gamma_j}(q) = \exp[ip_j(q - q_j)/\hbar] \psi_\gamma[x_j, \cos(\theta_j)(q - q_j)]$.

[†] The energy density for two degrees of freedom is $\mathcal{O}(\hbar^{-2})$.

If the system is bounded by a hard wall and we take this wall as the surface of section, ψ_γ is null there up to order $\sqrt{\hbar}^\dagger$. In the neighbourhood of a bouncing point, ψ_γ consists of two terms associated with the incoming and outgoing trajectory such that the combination satisfies boundary conditions [8] (see figure 1(b)). Then, working with its normal derivative we define

$$\varphi_\gamma(q) \equiv -\frac{i\hbar}{2m} \frac{\partial \psi_\gamma}{\partial \eta}(x, y)|_\zeta = \sum_j \dot{x}_j \cos(\theta_j) \psi_{\gamma_j}(q). \tag{11}$$

Moreover, $\hat{H}\varphi_\gamma(q) \equiv \hat{H}\psi_\gamma(x, y)|_\zeta$ for (10) and $\hat{H}\varphi_\gamma(q) \equiv -(i\hbar/2m)\partial \hat{H}\psi_\gamma/\partial \eta(x, y)|_\zeta$ for (11) are obtained to the leading order from (10) or (11) taking into account that

$$\hat{H}\psi_{\gamma_j}(q) \simeq g_\gamma[x_j, \cos(\theta_j)(q - q_j)] \psi_{\gamma_j}(q). \tag{12}$$

From now on $\varphi_\gamma(q)$ (equation (10) or (11)) is the object representing the resonance on the section. In order to describe an effective Hilbert space we define a norm on ζ ,

$$\langle \gamma | \gamma \rangle_\zeta \equiv \int_\zeta \varphi_\gamma(q)^* \varphi_\gamma(q) f(q) dq \tag{13}$$

such that (a)–(c) are satisfied. Then, evaluating the leading term of the integral in (13), it results in a classical criterion for specifying $f(q)$

$$T = \sum_{j=1}^m f(q_j) [\dot{x}_j \cos(\theta_j)]^{\mp 1}. \tag{14}$$

The signs (–) and (+) correspond to (10) and (11), respectively. We select a smooth real function $f(q)$, oscillating as slowly as possible, which satisfies (14) for all short POs required in the calculation. We note that the existence of $f(q)$ is not guaranteed for all sections. A hint to choose a section could be to make sure that the classical motion between consecutive points (the map) is simple. For example, in billiards the motion between bounces with the boundary is simple. On the other hand, eigenfunctions of billiards are reduced to the boundary in terms of their normal derivatives, and the metric on the boundary is defined by setting $f(q) = 2(\mathbf{r} \cdot \hat{\mathbf{n}})_{(q)}/\dot{x}^2$ [12], with $\hat{\mathbf{n}}$ the unit outgoing normal to the boundary and \mathbf{r} the position vector. Then, condition (14) (with sign (+)) reduces to

$$L = \sum_{j=1}^m 2(\mathbf{r} \cdot \hat{\mathbf{n}})_{(q_j)} \cos(\theta_j)$$

and this nice identity is valid for any PO in any billiard; the demonstration is a simple geometrical problem.

Now we can evaluate matrix elements over the section by using equations (10) or (11), (12) and (13) (δ crosses the section at q_k ($k = 1, \dots, m'$) with angles θ_k),

$$\langle \delta | \hat{O} \gamma \rangle_\zeta = \sum_{j,k=1}^{m,m'} A_{jk} \int_\zeta \psi_{\delta_k}(q)^* \hat{O} \psi_{\gamma_j}(q) f(q) dq. \tag{15}$$

$A_{jk} = 1$ for (10) and $A_{jk} = \dot{x}_j \dot{x}_k \cos(\theta_j) \cos(\theta_k)$ for (11). Defining $z_l = -i \cos^2(\theta_l) \Gamma(x_l)/2$, $c_l = q_l + i p_l/2z_l$ and $c_{jk} = (z_j c_j + z_k^* c_k^*)/(z_j + z_k^*)$, the Gaussian integrals in (15) are given to the leading order by

$$\int_\zeta \psi_{\delta_k}(q)^* \hat{O} \psi_{\gamma_j}(q) f(q) dq = \frac{D_{jk} f(c_{jk}) \exp[-B_{jk}/\hbar + i(\alpha_j - \alpha_k)]}{\sqrt{T_\gamma T_\delta \dot{x}_j \dot{x}_k |Q(x_j) Q(x_k)| (z_j + z_k^*)/J}}.$$

\dagger For billiards, the natural Poincaré surface of section is the boundary where ψ_γ is null (for Dirichlet conditions).

$B_{jk} = z_j z_k^* (c_j - c_k^*)^2 / (z_j + z_k^*) + p_j^2 / 4z_j + p_k^2 / 4z_k^*$ and $\alpha_l = S(x_l) / \hbar - \phi(x_l) / 2$. Moreover, $D_{jk} = 1$ for $\hat{O} = \hat{1}$ and $D_{jk} = g_\gamma[x_j, \cos(\theta_j)(c_{jk} - q_j)]$ for $\hat{O} = \hat{H}$. Then, equation (15) gives matrix elements in terms of classical quantities evaluated at the intersection points of the orbits with the surface of section.

Finally, in order to obtain the eigenenergies and eigenfunctions of a bounded chaotic Hamiltonian system in a given energy range, we proceed as follows. The family of resonances of the shortest periodic orbit γ_1 , living in the required energy range is constructed. The density of resonances associated with γ_1 is $\rho_1 \simeq T_1 / 2\pi\hbar$, with T_1 the period of γ_1 . Later, we do the same with the next shortest orbit γ_2 , and so on (using only primitive orbits). The process stops when the whole density of resonances equals the mean energy density ρ_E ,

$$T_H \equiv 2\pi\hbar \rho_E \simeq 2\pi\hbar \sum_{k=1}^{N_{po}} \rho_k \simeq \sum_{k=1}^{N_{po}} T_k. \quad (16)$$

Equation (16) is actually impressive, it shows that the number of POs, N_{po} , required in the calculation is very small and increases at most *linearly* with the Heisenberg time T_H . More precisely $N_{po} \simeq hT_H / \ln(hT_H)$, where h is the topological entropy. For methods related to the trace formula $N_{po} \simeq \exp(hT_H) / (hT_H)$.

Another interesting quantity is the number of resonances N_{res} contributing to one eigenfunction. This number is proportional to the mean dispersion (see (c)) and to ρ_E

$$N_{res} \simeq 2.6\hbar \rho_E \sqrt{\langle \lambda^2 \dot{x}^2 \rangle} \quad (17)$$

with $\langle \rangle$ the average over POs (using the factor 2.6 in (17), 99% of an eigenfunction is recovered because $\text{erf}(2.6/\sqrt{2}) \simeq 0.99$). Then, we select N resonances ($N \geq N_{res}$), consecutive in energy, and call them $\Gamma_1, \dots, \Gamma_N$. Later, by solving the following generalized eigenvalue problem:

$$\sum_{j=1}^N (\langle \Gamma_k | \hat{H} | \Gamma_j \rangle - E \langle \Gamma_k | \Gamma_j \rangle) \xi_j = 0 \quad \forall k \quad (18)$$

the eigenenergies E and eigenvectors ξ in the basis of resonances are given.

The main idea for the selection of resonances is to obtain a quasi-orthogonal basis of highly localized (in energy) functions. The best way of satisfying quasi-orthogonality is to use short periodic orbits. Now, for orbits with comparable periods we select the one with minimum energy dispersion (see (c)). This analysis works for hard chaos systems (where all POs are unstable and isolated). However, for systems with a fraction of regular motion, we need to include the same fraction of regular functions in the basis. And, for systems with a continuous family of neutral POs (e.g. the bouncing-ball family in the stadium billiard), a corresponding fraction of phase-space localized functions is required.

In conclusion, we need to construct adequate functions in each classically different region of phase space, with the number of them satisfying the required mean density (obtained semiclassically) in each region. In a chaotic region, functions (we call them resonances) are constructed with the shortest POs, and the number of them used to fill the Hilbert space is very small as implied in equation (16). In order to obtain the interaction between two given short POs it is possible to follow different strategies. Thinking at the classical level it is necessary to use at least a PO living in the neighbourhood of the previous ones. Then, to obtain all matrix elements, the period of the orbits required in the full calculation would be of the order of the Heisenberg time, and no advantage is reached with respect to other approaches. However, in this paper we showed that thinking at the quantum level, the interaction between short POs

can be evaluated simply in terms of transverse excitations. In this way, all the information required in the calculation is contained in short POs.

We would like to state some remarks.

- (a) This theory has been applied successfully to the desymmetrized stadium billiard [13].
- (b) The eigenvalues have an error $\mathcal{O}(\hbar)$ and eigenfunctions an error $\mathcal{O}(\sqrt{\hbar})$.
- (c) By the inclusion of transverse excitations in the construction of highly excited resonances we have obtained wavefunctions [14] with the hyperbolic structure characteristic of unstable periodic orbits [15].
- (d) The basis of resonances is particularly useful for parametric-dependent systems. In fact, this is the *adiabatic basis*, and is very difficult to find in chaotic systems [5].
- (e) Matrix elements between resonances from the same orbit (with different excitation numbers) are zero in this approximation. On the other hand, we expect that only one resonance from each family (associated with a single orbit), contributes to an individual eigenstate.
- (f) We should stress that, at present, it is not convenient to make a direct comparison between the efficiency of the trace formulae and this approach, which finally requires a diagonalization. Taking into account that the Hamiltonian in the resonance basis is sparsely connected, we hope to obtain an expression in the same spirit as the trace formula in the future.

Finally, within this formalism, we envisage a generic eigenfunction (in the extreme semiclassical limit) to be constructed with resonances of a large number of POs which cover the energy surface uniformly [16]. In this picture the intensities of the resonances would be described by a Gaussian distribution (depending on the difference between the energies of the corresponding resonances and the eigenfunction), plus fluctuations characteristic of the particular eigenfunction. Of course, this is not a new idea, the Gaussian distribution of intensities corresponds to the hypothesis by Berry [17] and Voros [18] of a uniform distribution of the eigenfunction (its Wigner representation) on the energy surface. On the other hand, the fluctuations are responsible for the scar phenomenon [19]. In this context, we would like to emphasize that this approach is very convenient for the study of fluctuations through a deep understanding of the degree of connection of the Hamiltonian matrix. At present we are working in that direction.

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